PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Terrestrial--Doppler Adjustment and Analysis of the Primary Triangulation of Great Britain: Preliminary Report [and Discussion]

V. Ashkenazi, S. A. Crane, J. W. Williams, J. D. A. Dean and D. W. Proctor

Phil. Trans. R. Soc. Lond. A 1980 **294**, 385-394
doi: 10.1098/rsta.1980.0046

Phil. Trans. R. Soc. Lond. A **294**, 385–394 (1980)
Printed in Great Britain

Terrestrial–Doppler adjustment and analysis of the primary triangulation of Great Britain: preliminary report

By V. Ashkenazi,† S. A. Crane,† J. W. Williams‡ and J. D. A. Dean‡

† *Department of Civil Engineering, University of Nottingham, University Park, Nottingham NG7 2RD, U.K.*

‡ *Ordnance Survey, Romsey Road, Maybush, Southampton SO9 4DH, U.K.*

The 1977 readjustment of the primary triangulation of Great Britain resulted in an internally consistent geodetic framework. The remaining suspected sources of systematic error could only be detected through comparisons with satellite Doppler derived observations carried out at selected stations of the network. These comparisons are followed by a simultaneous adjustment and strength analysis involving both terrestrial observations and Doppler positional data. The results of this combined adjustment are compared with the 1977 coordinates to assess the contribution of the satellite Doppler-derived data.

Details are given of the choice of the reference system, the treatment of the Doppler data and the assignment of *a priori* standard errors of the various types of observations. The variance–covariance analysis is carried out with real data as well as with simulated observations in order to quantify the contribution of additional satellite Doppler observations to the geometrical strength of the combined network.

1. Introduction

1.1. *OSGB Scientific Networks*

There have been two triangulations of Great Britain. The second of these, the Retriangulation, with which this paper is concerned, was carried out with the main aim of providing urgently needed control for mapping at 1:1250 scale and was adjusted in seven main blocks. The system of coordinates on the Airy spheroid produced from this adjustment is referred to as Ordnance Survey of Great Britain 1936 (OSGB 36). For cartographic reasons the work was adjusted by Hotine in a way designed to minimize shifts between old and new coordinates. It was known that the scale was too large, and subsequent checks on two baselines indicated that the discrepancy was in the order of 15 parts/10^6, but was probably consistent throughout the country. However, later measurements with microwave and light wave e.d.m. instruments showed that the error varied from as much as 48 parts/10^6 in parts of Scotland to 1 part/10^6 in other areas.

A rigorous and simultaneous readjustment of the Retriangulation was carried out in 1970 by the Ordnance Survey of Great Britain in cooperation with the Department of Civil Engineering at the University of Nottingham (Ashkenazi *et al.* 1972). The resulting coordinates, which formed the Scientific Network 1970 (OSGB 70(SN)), constituted a significant improvement over the previous OSGB 36 coordinates. The average *a posteriori* standard errors of a side were 2.4 parts/10^6 in length and 0.5″ in azimuth.

Since 1970 some extra distances and Laplace azimuths have been measured, mainly in conjunction with the Edinburgh–Malvern–Dover precise traverse. Moreover, the observation of geoidal sections between 1970 and 1977 allowed the production of a geoid contour map on

OSGB 70 (SN) datum as the basis for distance reduction to the spheroid. A new preliminary scientific adjustment was carried out by the Ordnance Survey in 1977 (OSGB 77) including this correction. However, it is not yet the Ordnance Survey's intention to publish new scientific coordinates to replace those of OSGB 70.

1.2. *U.K. Doppler observations*

The period 1970–7 also saw major advances in satellite Doppler positioning which, in turn, led to the opinion that geodetic positions derived from such observations could be used to strengthen terrestrial networks. The good geometry of, and the abundance of scale control in, the U.K. network meant that it was ideal for a series of tests in which terrestrial and satellite observations could be compared and eventually combined, both as a means of determining systematic errors, and to investigate the practicability of such a procedure.

Satellite Doppler positions were available at 25 stations of the primary network. These were obtained as follows: (*a*) seven stations observed by 512 Specialist Team Royal Engineers (S.T.R.E.), mainly in 1976–7, with a Geoceiver; (*b*) four stations observed in the 1976 U.K. Doppler campaign, with a Marconi CMA-722B; (*c*) five stations observed by Decca Survey in 1978, with a JMR-1; (*d*) nine stations observed in the 1978 U.K. Translocation Doppler campaign, along the precise traverse, with a JMR-1.

After a consideration of the quality and availability of the Doppler data, only the stations in (*a*) and (*d*) above, a total of 16, were used in the comparisons and the combined adjustment (see figure 1). The geocentric coordinates of all these stations were computed by single point positioning in the NWL 9D system, with the use of U.S. DMA's precise ephemeris. The coordinates of the S.T.R.E. stations were supplied through the Ordnance Survey, while those of the traverse stations were computed by using the Nottingham UNDAP program.

2. Translation parameters Doppler–OSGB 77

To bring Doppler positions computed in the NWL 9D system into a properly defined geodetic reference system, it is necessary to apply two corrections, one for scale and one for longitude rotation. At present (October 1978), the accepted values are -0.4 part/10^6 and $0.8''$ respectively (Anderle 1978; Hothem *et al.* 1978). This scaled and rotated system will be referred to in this paper as the Doppler 78 datum. Geocentric Cartesian coordinates in this system would then only require three translation parameters, ΔX, ΔY and ΔZ, to transfer them into any other national or continental geodetic reference system.

The translation parameters required to transform the Doppler 78 coordinates of the 16 stations into the OSGB 77 system were obtained by averaging the geocentric Cartesian coordinate differences between the Doppler 78 coordinates and the corresponding Cartesian coordinates in the terrestrial system. Mean values were calculated for the two sets of Doppler stations and are given in table 1. Because of a suspected (and later confirmed) scale bias in the terrestrial network, translation parameters were also computed between Doppler 78 and the scale corrected OSGB 78 network, in which all the (tellurometer measured) distances had been scaled up by 2.6 parts/10^6 (§3.1). These are given in table 2. An examination of tables 1 and 2 shows that there is no 'significant' difference in the values of the translation parameters between the two sets of Doppler stations or the two models of the terrestrial network. The overall mean values of these parameters for the OSGB 77 network, rounded to the nearest 0.5 m, were thus

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

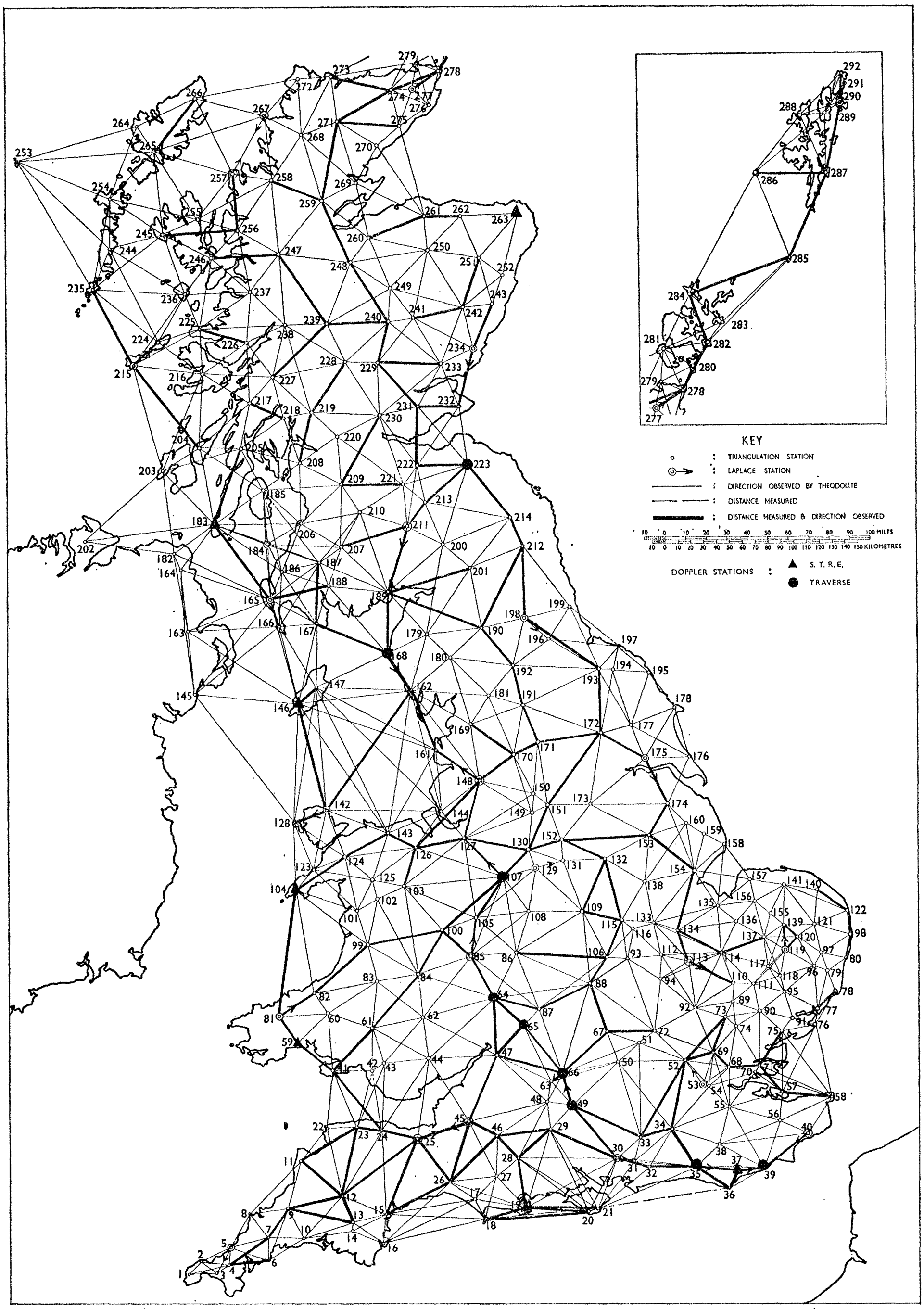

FIGURE 1. The Scientific Network 1978.

adopted and used subsequently for the transformation of the Doppler 78 coordinates into the OSGB system. These are $\Delta X = -369.0$ m; $\Delta Y = +111.5$ m; $\Delta Z = -430.0$ m.

It is estimated that these values have an (external) standard error smaller than 1 m. Numerical tests carried out at Nottingham have shown that this level of accuracy is adequate to prevent any significant errors in the corresponding relative positions of the Doppler stations, expressed in terms of spheroidal distances and azimuths.

Table 1. Mean translation parameters from Doppler 78 to OSGB 77

	ΔX/m	ΔY/m	ΔZ/m
S.T.R.E. only	-368.52 ± 0.33	111.95 ± 0.28	-430.30 ± 0.35
traverse only	-369.33 ± 0.23	111.21 ± 0.31	-429.66 ± 0.30
all stations	-368.98 ± 0.22	111.54 ± 0.23	-429.94 ± 0.24

Table 2. Mean translation parameters from Doppler 78 to OSGB 78

	ΔX/m	ΔY/m	ΔZ/m
S.T.R.E. only	-368.95 ± 0.27	111.58 ± 0.20	-430.04 ± 0.33
traverse only	-369.58 ± 0.26	111.12 ± 0.26	-429.49 ± 0.25
all stations	-369.30 ± 0.20	111.27 ± 0.19	-429.73 ± 0.21

3. Scale and orientation comparisons

3.1. *Chord distances*

Chord distances are independent of rotation and translation parameters and constitute therefore a valuable means of comparing the accuracies of relative positions of stations in two different coordinate systems, in this case the Doppler 78 and OSGB datums. Chord distances were computed in both systems of all combinations of pairs of points. The precise traverse and S.T.R.E. stations were considered separately, as two subsets of 36 and 21 lines respectively (table 3). The results, in terms of the mean difference between terrestrial and Doppler 78 distances in parts/10^6 are given in table 4.

Table 3. Number of test lines in distance and azimuth comparisons

	Doppler stations	
test lines	S.T.R.E.	traverse
all lines	21	36
lines over 300 km	14	13

Table 4. Mean chord distance differences, Doppler 78 – terrestrial (parts/10^6)

terrestrial network	test lines	Doppler stations: S.T.R.E.	Doppler stations: traverse
OSGB 77	all lines	$-2.15 \pm 0.43''$	$-2.48 \pm 1.47''$
	lines over 300 km	$-2.88 \pm 0.40''$	$-3.22 \pm 0.43''$
OSGB 78	all lines	$-0.16 \pm 0.45''$	$-0.97 \pm 1.47''$
	lines over 300 km	$-0.64 \pm 0.45''$	$-1.33 \pm 0.39''$

As one would expect, the accuracy of a single station Doppler fix (1 m) leads to relatively large discrepancies in comparisons involving short lines, but much smaller and consistent differences in the case of long lines. Consequently, it was decided to calculate the average values of these differences for lines over 300 km in length as well as for all lines (table 4). Clearly, the results indicate a significant scale discrepancy of about 3 parts/10^6 between Doppler 78 and OSGB 77, the latter being smaller. The Ordnance Survey had suspected this for some time (Richards 1976), attributing it to the number of microwave distances in the network. Earlier results from a large number of test lines had revealed a systematic discrepancy of 2.6 parts/10^6 between measurements with microwave e.d.m. on the one hand, and visible

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

light wave e.d.m. on the other, with the former being shorter. This was also confirmed by results obtained in Australia (Bomford 1973).

Consequently, a second model of the OSGB network (OSGB 78) was proposed, based on the OSGB 77 observations, but with all the microwave distances scaled up by 2.6 parts/10^6. Chord distance comparisons carried out for this network (table 4) show that the mean scale difference between the Doppler and terrestrial systems has now been reduced to about 1 part/10^6. If the scale of the NWL 9D system is accurate to 0.1 part/10^6 (as claimed by Hothem *et al.* 1978) this indicates a residual systematic error in the terrestrial network, though this only amounts to approximately 1 m along its entire length.

3.2. *Spheroidal distances and azimuths*

A similar comparison was also carried out by using spheroidal distances and azimuths. Doppler coordinates were transformed, with the use of the parameters derived in §2 and reduced to the (Airy) ellipsoid which is the reference for the OSGB coordinates. This was followed by the computation of spheroidal azimuths and distances for both sets of coordinates, and the averaging of the differences obtained: the results for distances and azimuths are given in tables 5 and 6 respectively, for both models of the terrestrial network (OSGB 77 and OSGB 78).

TABLE 5. MEAN SPHEROIDAL DISTANCE DIFFERENCES, DOPPLER 78 – TERRESTRIAL (PARTS/10^6)

terrestrial network	test lines	Doppler stations: S.T.R.E.	Doppler stations: traverse
OSGB 77	all lines	-2.15 ± 0.42	-2.49 ± 1.47
	lines over 300 km	-2.88 ± 0.39	-3.19 ± 0.42
OSGB 78	all lines	-0.15 ± 0.43	-0.96 ± 1.47
	lines over 300 km	-0.83 ± 0.38	-1.29 ± 0.38

TABLE 6. MEAN SPHEROIDAL AZIMUTH DIFFERENCES, DOPPLER 78 – TERRESTRIAL

terrestrial network	test lines	Doppler stations: S.T.R.E.	Doppler stations: traverse
OSGB 77	all lines	$0.16 \pm 0.12''$	$0.52 \pm 0.19''$
	lines over 300 km	$0.06 \pm 0.09''$	$0.46 \pm 0.14''$
OSGB 78	all lines	$0.11 \pm 0.11''$	$0.54 \pm 0.19''$
	lines over 300 km	$0.03 \pm 0.09''$	$0.46 \pm 0.14''$

As expected, the spheroidal distance comparisons are in close agreement with those for the chord distances. On the other hand, differences for the azimuths show an unexpected and significant difference between the two sets of Doppler data: while the azimuths of the lines connecting the S.T.R.E. stations agree well with those of the OSGB network, there appears to be a systematic discrepancy of about 0.5″ between the Doppler azimuths and those of the traverse stations.

However, in contrast to the S.T.R.E. stations, which are located along the edges of the OSGB network, the traverse stations are very strongly controlled in orientation, through the observation of frequent astronomical azimuths. Clearly, there is an anomaly which requires an explanation. Either the astronomical azimuths observed along the traverse have a small systematic observational error or there is a corresponding error in the 1978 Doppler system, due to the different Doppler receivers or reduction program used.

4. Best fit between networks

As a matter of academic interest, it was decided to find out how closely the Doppler 78 and OSGB 78 systems could be brought into agreement, by a least squares Helmert transformation involving five parameters (scale, longitude rotation and three geocentric datum shifts), using all 16 stations. The results are given in table 7. The resulting scale parameter of about 1 part/10^6 is in agreement with the discrepancy already observed after correcting the microwave distances. However, together with the longitude rotation of 0.15″, this leads to three translation parameters whose values differ by up to 6 m from the previously adopted values. These results show that further systematic differences exist between Doppler 78 and OSGB 78, though at present these cannot be explained.

Table 7. 'Best fit' Helmert transformation parameters, Doppler 78 to OSGB 78

scale	μ =	-1.2 ± 0.8 parts/10^6
longitude rotation	ϕ_z =	$-0.15 \pm 0.18''$
translation	ΔX =	-364.5 ± 3.0 m
	ΔY =	113.9 ± 3.5 m
	ΔZ =	-423.6 ± 3.9 m

Table 8. Mean residuals (metres) after 'best fit' transformation

	Doppler stations	
	S.T.R.E.	traverse
latitude	-0.42 ± 0.20	$+0.36 \pm 0.15$
longitude	$+0.17 \pm 0.15$	-0.23 ± 0.24
spheroidal height	-0.02 ± 0.42	-0.01 ± 0.32

Table 9. Mean differences in spheroidal azimuth and distance 'best fit' Doppler–OSGB 78

		Doppler stations	
	test lines	S.T.R.E.	traverse
azimuth differences	all lines	$0.00 \pm 0.11''$	$0.42 \pm 0.19''$
	lines over 300 km	$-0.09 \pm 0.08''$	$0.36 \pm 0.15''$
distance differences (parts/10^6)	all lines	1.06 ± 0.43	0.25 ± 1.47
	lines over 300 km	0.38 ± 0.38	-0.07 ± 0.38

The residual differences between the two sets of station coordinates after applying the 'best fit' Helmert transformation parameters to the Doppler coordinates are given in table 8. Once again, there appears to be a discrepancy between the two subsets of Doppler stations, S.T.R.E. and traverse.

The results of a comparison of spheroidal distances and azimuths between the 'best fit' Doppler and OSGB 78 (table 9) show that, apart from the already known scale difference, the

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

new transformation parameters lead to only a minor improvement in the correspondence between the two networks. This is not surprising in the case of the small area covered by the OSGB network, where a longitude rotation can be almost exactly compensated by a small change in ΔY.

5. Preliminary combined adjustment

In order to assess the effect of incorporating Doppler observations in a terrestrial network, an adjustment and error analysis was carried out, by using six models of the U.K. Primary Triangulation, for both the OSGB 77 and 78 networks:

model 1: OSGB 77, no Doppler stations;
model 2: OSGB 78, no Doppler stations;
model 3: OSGB 77, 16 Doppler stations (positional s.e. 0.5 m);
model 4: OSGB 78, 16 Doppler stations (positional s.e. 0.5 m);
model 5: OSGB 77, 16 Doppler stations (positional s.e. 1.0 m);
model 6: OSGB 78, 16 Doppler stations (positional s.e. 1.0 m).

The Doppler derived positions, transformed into the OSGB system, were incorporated in the adjustment as observed position equations in terms of latitude and longitude.

The comparisons between the results obtained for the different models were carried out over a sample of 30 test lines, distributed throughout the network and grouped as follows:

15 short lines (under 100 km);
5 medium lines (100–300 km);
5 long lines (300–500 km);
5 very long lines (over 500 km).

The distance and azimuth (with corresponding standard errors) were computed for all these lines in each model. The results obtained (table 10) indicate that there is little to choose between the models. The standard errors of the test lines seem barely affected by the inclusion of the Doppler stations. This is not completely unexpected, as 32 extra position observations over and above the 2000 terrestrial observations would have little effect on the normal matrix, and hence on the variance–covariance matrix as a whole, unless they are assigned artificially high

Table 10. Effects of incorporating Doppler observations in OSGB terrestrial network

	adjustment model					
	1	2	3	4	5	6
mean change in test line lengths from model 1 (parts/10^6)	—	+2.10	+0.39	+2.28	+0.12	+2.16
mean change in test line azimuths from model 1	—	−0.02″	−0.12″	−0.11″	−0.06″	−0.07″
mean s.e. of short test line lengths (parts/10^6)	2.62	2.60	2.67	2.62	2.61	2.58
mean s.e. of short test line azimuths	0.48″	0.47″	0.46″	0.45″	0.46″	0.46″
mean shift in observed Doppler position/m	—	—	0.83	0.66	0.90	0.67
σ_0	0.9637	0.9553	0.9836	0.9644	0.9611	0.9495

weights. The effect on the actual distances and azimuths of the test lines is similar. For instance, in the case of the OSGB 77 network, which has been shown to have a scale bias of 3 parts/10^6, the introduction of 16 observed Doppler stations, with 0.5 m standard error in each coordinate, causes only a reduction of about 0.4 part/10^6 in that discrepancy. The corresponding effect on azimuths with a known systematic bias of 0.5″, is a correction of only 0.1″. The Doppler observations, in turn, are forced, as a result of the adjustment, to undergo large distortions, as expressed by residuals, of about 0.5 m, their *a priori* standard errors making hardly any difference.

Clearly, the 16 observed Doppler stations do not seem to overcome the systematic scale and orientation errors in such a strong and compact terrestrial network which incorporates a large number of observed distances and azimuths. The model used for the adjustment does not appear to allow Doppler observations to play their proper part and make their contribution to the strengthening of the network.

One solution (probably the easiest) to overcome this difficulty would be to use the observed Doppler positions only to determine the systematic scale and orientation errors in the terrestrial network, to apply these corrections to the terrestrial observations, and then proceed with the adjustment only with the 'Doppler corrected' terrestrial data.

A more elegant solution would be provided by using different adjustment models. The use of a scale correction factor for measured distances, to be determined as an unknown in the combined adjustment would not, on its own, solve the problem. Again, the relative *a priori* standard errors of Doppler and terrestrial observations would not allow this factor to emerge at anywhere near its correct value. The same applies to unknown 'orientation' parameters. Clearly, the solution lies with the proper modelling of the *a priori* variance–covariance matrix. Even an *a priori* standard error of 0.5 m for a Doppler fix is not adequate, unless corresponding inter-station covariances can be estimated and introduced into the adjustment. This should be done not only for 'short arc' solutions, but also for precise ephemeris computed 'single point' positions. Furthermore, one should experiment with adjustment models involving unknown translation parameters as well as unknown scale factors and additive corrections for the terrestrial distances and azimuths respectively.

In a combined adjustment one is less interested in the absolute geocentric positional accuracy of the Doppler fix than in the relative positional accuracy of two or more stations. As an illustration, one could enter two observed Doppler fixes into an adjustment as an observed 'long' distance and azimuth with appropriate standard errors. Of course, it could be difficult to use this procedure with three or more observed Doppler fixes. Only a full covariance matrix would provide the proper solution.

6. Conclusions

(1) Satellite Doppler observations have confirmed a suspected scale bias of about 3 parts/10^6 in the OSGB 77 terrestrial network. Most of this bias is accounted for by the preponderance of microwave distances in the network.

(2) Similarly, an azimuth discrepancy of 0.5″ appears to exist in part of the OSGB 77 Network. This is less consistent and has not yet been explained.

(3) The incorporation of Doppler observations in geometrically strong terrestrial networks in a combined adjustment *with a standard model* is not effective and contributes little to the overall strength of the network.

(4) A simple way of using Doppler positions would be in a mode of comparison, for finding systematic scale and orientation errors in the terrestrial network, subsequently leaving them out of the combined adjustment.

(5) However, the establishment and use of an appropriate *a priori* covariance matrix is the only theoretically sound method of using Doppler derived position equations in conjunction with terrestrial observations.

(6) Moreover, one should test adjustment models involving unknown scale and orientation corrections to the terrestrial observations.

The work leading to this paper was carried out jointly with the Ordnance Survey of Great Britain and a research team from the Department of Civil Engineering of the University of Nottingham under the direction of Professor R. C. Coates.

Research on satellite Doppler techniques at Nottingham is carried out with the financial support of the United Kingdom Science Research Council and Decca Survey Limited.

References (Ashkenazi *et al.*)

Anderle, R. J. 1978 Mean Earth's ellipsoid based on Doppler satellite observations. Presented at Spring Meeting of the American Geophysical Union.

Ashkenazi, V. 1977 RETrig and EDOC: European terrestrial and Doppler networks. Presented at 35th J.L.G. Meeting, Luxemburg.

Ashkenazi, V., Cross, P. A., Davies, M. J. K. & Proctor, D. W. 1972 The readjustment of the retriangulation of Great Britain and its relationship to the European terrestrial and satellite networks. *Ordnance Surv. prof. Pap.* no. 24.

Ashkenazi, V. & Cross, P. A. 1975 Strength of long lines in terrestrial geodetic control networks. Presented at XVI General Assembly of the I.A.G., Grenoble.

Ashkenazi, V., Gough, R. J. & Sykes, R. M. 1977 Preliminary analysis of the United Kingdom Doppler campaign. Presented at the 35th J.L.G. Meeting, Luxemburg.

Ashkenazi, V. & Richards, M. R. 1976 A proposed investigation on absolute and relative accuracies in terrestrial positioning by satellite-Doppler methods. *Chart. Surv.* **3** (4). no. 4.

Ashkenazi, V. & Sykes, R. M. 1978 Precise and broadcast ephemerides position determination over the U.K. and Europe. Presented at Second International Symposium on the Use of Artificial Satellites in Geodesy and Geodynamics, Athens.

Bomford, A. G. 1973 Geodetic models of Australia. Presented at I.A.G. Symposium on Computational methods in Geometrical Geodesy, Oxford.

Cross, P. A. 1970 Strength analysis of large geodetic triangulation networks. Ph.D. thesis, University of Nottingham.

Hothem, L. D., Robertson, D. S. & Strange, W. E. 1978 Orientation and scale of satellite Doppler results based on combination and comparison with other space systems. Presented at Second International Symposium on Problems Related to the Redefinition of North American Geodetic Networks, Washington D.C.

Kouba, J. 1978 Datum considerations for test adjustments of Canadian primary horizontal geodetic network. Presented at Second International Symposium on Problems Related to the Redefinition of North American Geodetic Networks, Washington D.C.

Parm, T. 1976 *High precision traverse of Finland.* Helsinki: Finnish Geodetic Institute.

Richards, M. R. 1976 Relating Doppler datum to Ordnance Survey Datum. Presented at International Geodetic Symposium on Satellite-Doppler Positioning, Las Cruces.

Schlüter, W., Seeger, H., Soltau, G., Wilson, P., Wolf, P., Pesec, P. & Rinner, K. 1978 The present status of the German–Austrian Doppler Observation Campaign (DÖDOC). Presented at Second International Symposium on the Use of Artificial Satellites for Geodesy and Geodynamics, Athens.

Weightman, J. A. 1975 Doppler ties to European Datum and the European Geoid. Presented at Commonwealth Survey Officers Conference, Cambridge.

Discussion

D. W. Proctor (*Directorate of Overseas Surveys, Kingston Road, Tolworth, Surbiton, Surrey KT5 9NS, U.K.*). Mr Crane has given us the impression that he is concerned that the introduction of

PHILOSOPHICAL TRANSACTIONS OF THE ROYAL SOCIETY A MATHEMATICAL, PHYSICAL & ENGINEERING SCIENCES

Doppler stations, in various adjustments of the British triangulation network, did not cause any appreciable changes. I do not share his concern. I should have been very concerned to see material changes forced upon so strong a network by a few Doppler points. The Doppler would have to be grossly overweighted if it were allowed to distort the network. He gave weights based on assumed standard deviations of 1 m and 0.5 m, which seem sensible estimates, and finished with residuals of the same order, which is only to be expected. This only demonstrates that the apparent differences between the two systems in azimuth and scale are probably not statistically significant; this is not surprising since with accuracies in the 0.5–1 m threshold Doppler cannot improve on 0.5″ in azimuth or 1 part/10^6 in scale until relative spacings are larger than is possible in a small network.

If the internal network strength had caused residuals at the Doppler points much larger than their assumed standard deviations, then one would have deduced a systematic disagreement of some significance. As it is, the situation seems entirely satisfactory and one for gratification on the part of both Ordnance Survey and Nottingham University.

S. A. Crane. As Mr Proctor says, the Ordnance Survey Primary Triangulation is a very strong network, which is well observed and rigorously adjusted. However, this does not preclude the possibility of systematic scale and orientation errors. One is indeed concerned about the results of direct comparisons of distances and azimuths of lines derived from Doppler positions, on the one hand, and computed from adjusted terrestrial coordinates, on the other. There is clearly a difference between the two sets, which is especially significant in terms of the well determined standard error of the scale of the Doppler system. The magnitude of this difference appears to correspond to the well known proportional difference between microwave and light wave measured distances. This is particularly illustrated along the two halves of the Edinburgh–Malvern–Dover traverse.

Clearly, only correct mathematical modelling of the adjustment – one which accounts for the covariances between the Doppler derived positions – is likely to enable the latter to make their proper contribution in reducing any systematic scale and orientation errors that may exist in the terrestrial data. It may turn out that even this would hardly affect the OSGB network, because of its relatively small size. But the knowledge gained from these tests could be put to use in very large national and continental networks that do not have such a strong configuration.